Thought Question

At a doctor's visit you are told that you have tested positive for a rare kidney disease. The doctor tells you that the test is very accurate and that more than 99% of people with the disease test positive. He suggests you immediately begin a course of treatment, which should cure the disease but has some serious side-effects. Before agreeing to the treatment, what additional information do you want to get from the doctor?

Base rates: Kidney example

	Disease	**No disease**		
Test Positive	99	?	?	positive test = disease? 99/??? = (??)
Test Negative	1	?	?	
	100	?		
	disease= positive test? 99/100 (**99%**)			

Need to worry about test? Need to know **base rate and false alarm rate.**

Base rates: Kidney example

	Disease	**No disease**		
Test Positive	99	1	100	positive test = disease? 99/100= (99%)
Test Negative	1	99	100	
	100	100		
	disease= positive test? 99/100 (**99%**)			

False Alarm Rate = 1%; Base rate: 50%
Need to worry about test? **YES**.

Base rates: Kidney example

	Disease	**No disease**		
Test Positive	99	990	1089	positive test = disease? 99/1089= (**9%**)
Test Negative	1	8910	8911	
	100	9,900		
	disease= positive test? 99/100 (**99%**)			

False Alarm Rate = 1%; Base rate: 1%
Need to worry about test? **only a little…**

Base rates: Kidney example

	Disease	**No disease**		
Test Positive	99	0	99	positive test = disease? 99/99= (**100%**)
Test Negative	1	9900	9901	
	100	9,900		
	disease= positive test? 99/100 (**99%**)			

False Alarm Rate = 0%; Base rate: 1%

Need to worry about test? **only a little…**

Base rates: Kidney example

	Disease	**No disease**		
Test Positive	99	50	149	positive test = disease? 99/149= (66%)
Test Negative	1	50	151	
	100	100		
	disease= positive test? 99/100 (**99%**)			

False Alarm Rate = 50%; Base rate: 50%

Need to worry about test? **MAYBE**

Problem Solving

What is a problem?

Dear Abby:
I have no shortage of (Facebook) friends, and new ones come in regularly. But some of the online invitations for friendship come from people I don't know. Some are from folks in my field, so there's some kinship and common bond even though we haven't formally met. Some are from college students, studying the field that I'm in (new media, grassroots media). Some seem to have no connection to me whatsoever, so those get treated like spam and the invitations rejected. But what's the etiquette here? Should I "befriend" complete strangers if we've got something in common? Should I limit my Facebook friends to people I've met? What to do, Abby?

"ripped from the headlines" 08-2007

The parts of a problem

What is a problem?

A problem has:

- Givens
- A goal
- Obstacles
- Operators

What is a problem?

A problem has:

- Givens:

  The initial conditions, objects, and information available.
- A Goal
- Obstacles
- Operators

What is a problem?

A problem has:

- Givens
- A Goal:

  The desired outcome or information.

  (No goal? What's the point?)
- Obstacles
- Operators

What is a problem?

A problem has:

- Givens
- A Goal
- Obstacles

  What is between givens and goal.

  (Without obstacles, you don't have a problem to solve…)
- Operators

What is a problem?

A problem has:

- Givens
- A Goal
- Obstacles
- Operators

  Means of transforming the conditions.

  (Without these, you can't solve the problem…)

Types of problems

well-defined problems

and

poorly defined (ill-defined) problems

Types of problems

well-defined problems

- components of the problem are clearly specified

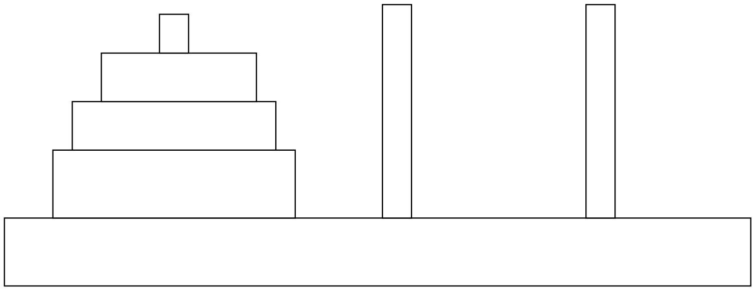
examples: games, math

poorly defined (ill-defined) problems

- uncertainty in givens, obstacles, operators, even goal

examples: health care decisions, managing friendships

A Classic Well-defined Problem



The Tower of Hanoi Problem

Move stack to far right peg, one disc at a time
A disc cannot be placed on a smaller disc

Let’s try it!

- http://www.mazeworks.com/hanoi/index.htm

Fun facts:

The Tower of Hanoi (sometimes referred to as the Tower of Brahma or the End of the World Puzzle) was invented by the French mathematician, Edouard Lucas, in 1883. He was inspired by a legend that tells of a Hindu temple where the pyramid puzzle might have been used for the mental discipline of young priests. Legend says that at the beginning of time the priests in the temple were given a stack of 64 gold disks, each one a little smaller than the one beneath it. Their assignment was to transfer the 64 disks from one of the three poles to another, with one important provision that a large disk could never be placed on top of a smaller one. The priests worked very efficiently, day and night. When they finished their work, the myth said, the temple would crumble into dust and the world would vanish.

Why are we still here then?

The number of separate transfers of single disks the priests must make to transfer the tower is 2 to the 64th minus 1, or 18,446,744,073,709,551,615 moves! If the priests worked day and night, making one move every second it would take slightly more than 580 billion years to accomplish the job!

A Classic Problem

- What are the givens?
  - Three disks of different sizes
  - Three poles
  - Initial state: three disks all stacked in order of size (smallest on top) on the left-most pole

* state: current description of the elements of the problem

A Classic Problem

- What is the goal?
  - Final state: Get all three disks stacked in order of size (smallest on top) on right-most pole.

A Classic Problem

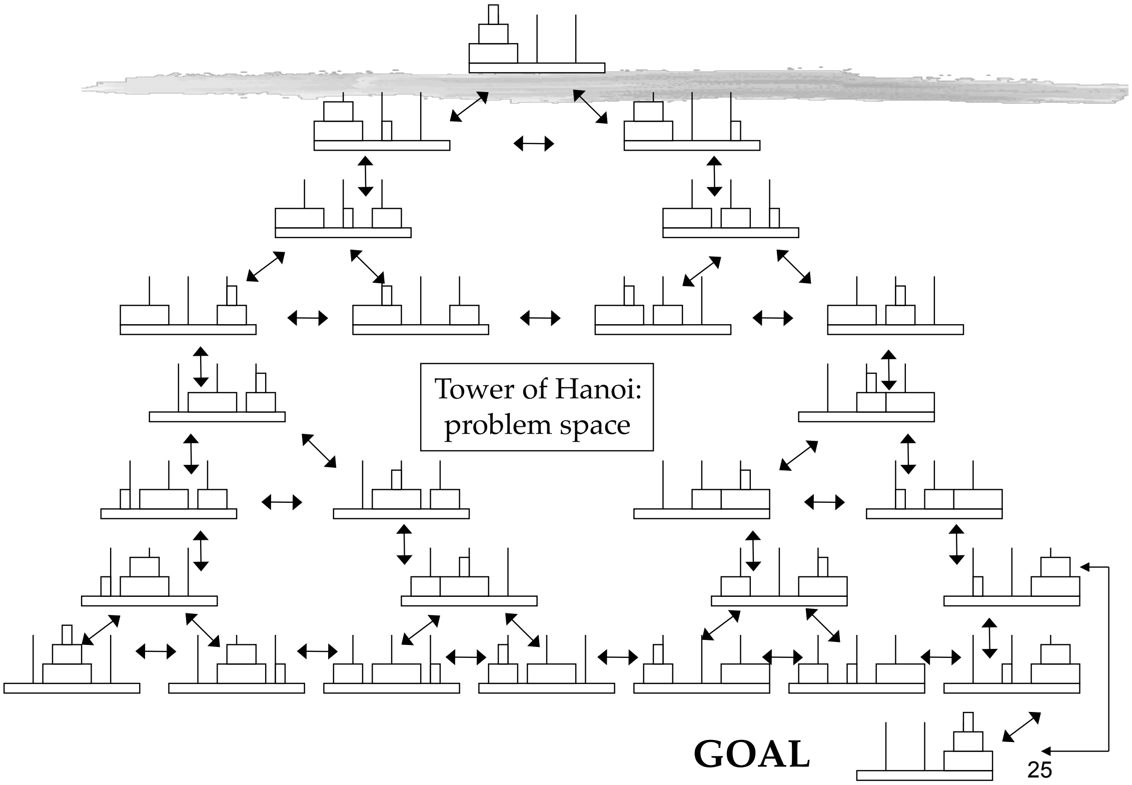
- What are the obstacles? (what is the difference between the current state and the goal)
  - We are on the wrong pole!

A Classic Problem

- What are the operators ("legal moves")?
  - Can move disks from one pole to another.
  - RESTRICTIONS:
    - can move only one disk at a time
    - can only move the top disk
    - can only put disk onto empty pole or onto one having a *larger* disk
- As you apply the operators, you create intermediate states of the problem.

Problem space

- In well-defined problems, we know:
  - what the initial state is
  - what the final state should be
  - what all the allowed moves are
- Therefore: in principle, we can make a map, called a problem space, that shows all the possible moves and trajectories ...



Problem space

- With a problem space worked out, you can determine the most efficient mapping from the initial state to the goal state.
- For the Tower of Hanoi, can do it in seven moves.
  - Can have an objective measure of problem solving efficiency and of the value of any given move.

Studying problem solving

verbal protocols:

subjects are asked to "think aloud" during the course of solving a problem

intermediate products:

analyze the steps the subjects take when working towards the goal

computer simulations:

program a computer to follow possible routes to a problem solution

The Newell and Simon approach

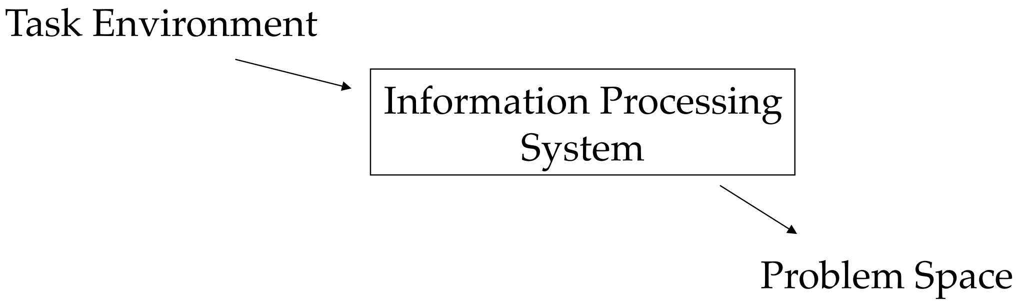
Problem Solving as Representation and Search:

information processing approach to problem solving

- Have resource limitations
    - STM capacity
    - Time to access LTM
    - Time to store info into LTM
- Processing is serial
- Representations are discrete
    - Goal, operators, etc.

(1) Representation:

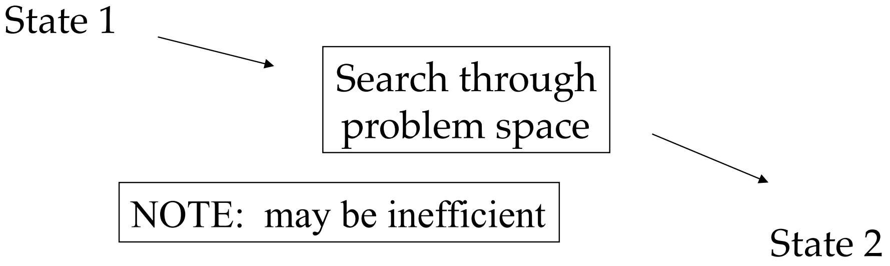
Information processing system acts on the task environment (the problem in the "real world") to create a problem space



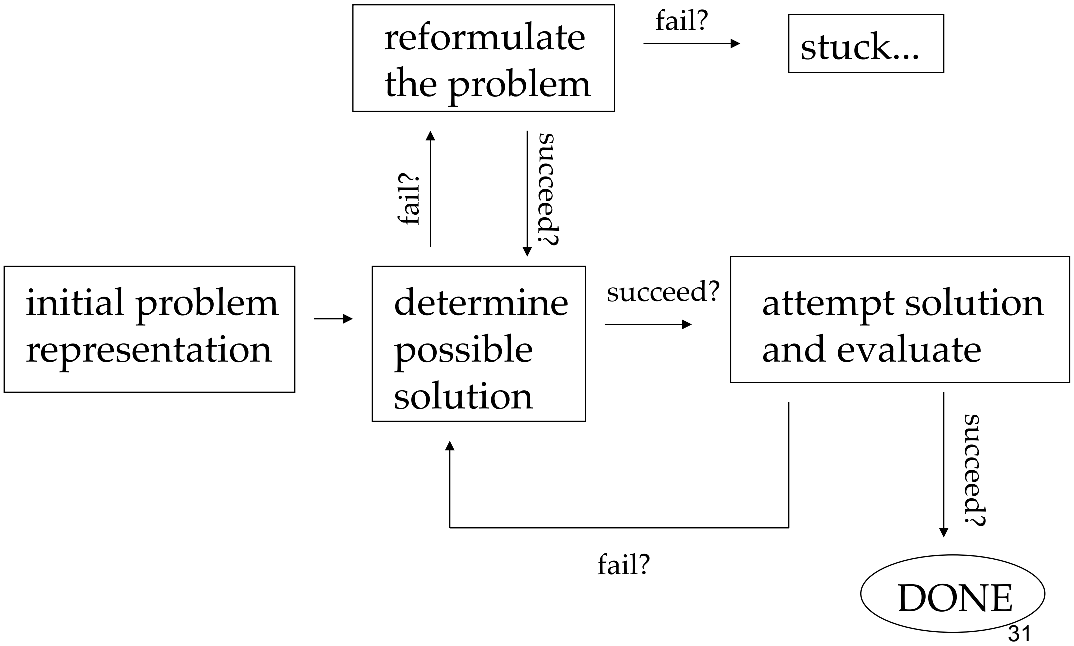
NOTE: may be incomplete

(2) Search:

Information processing system uses operators to move through the problem space, "searching" for the next move



NOTE: may be inefficient



reformulate the problem

fail?

stuck...

fail?

succeed?

initial problem representation

determine possible solution

succeed?

attempt solution and evaluate

succeed?

fail?

DONE

Two “kinds” of search

Algorithmic: systematic procedure that is guaranteed to lead to correct solution (all possibilities are considered)

+ = guaranteed solution (complete search)

- = time consuming, often too many possibilities

Heuristic: “shortcut” method of guiding search so that the solution is likely

+ = easier to process

- = no guarantee of a solution

Algorithms

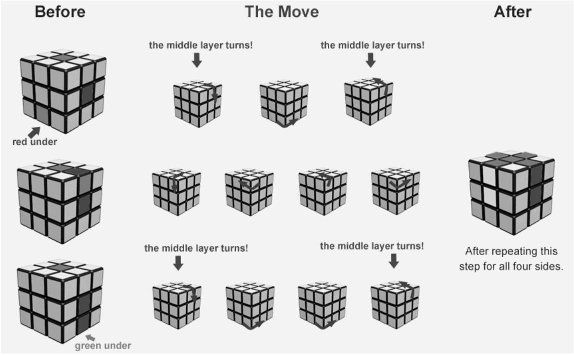
- Good algorithm:
  - Approach to a problem that solves the problem each and every time
  - Not always the most efficient
  - Something that a computer could do for you

Example Algorithm

- Example algorithm for the tower of Hanoi
  - *(this is something you could implement in a computer)*
  - Step 1 – Move n-disk tower from source to destination via resting place
  - Step 2 – Move n-1 disk tower from source to resting place
  - Step 3 – Move 1 disk tower from source to destination
  - Step 4 – Move n-1 disk tower from resting place to destination

Example Algoritm

- The Rubicks cube
  - well-defined problem: one solution
  - algorithm is series of moves to apply to get to the solution from starting state



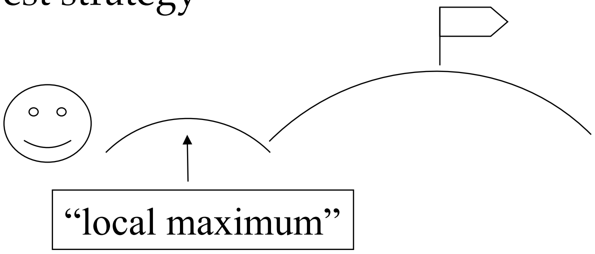
Some heuristics

Simple search

- assess current state and possible next states
- choose one that moves you closest to the goal
- "Hill climbing"

PROBLEM?

Sometimes temporarily moving away from goal is the best strategy



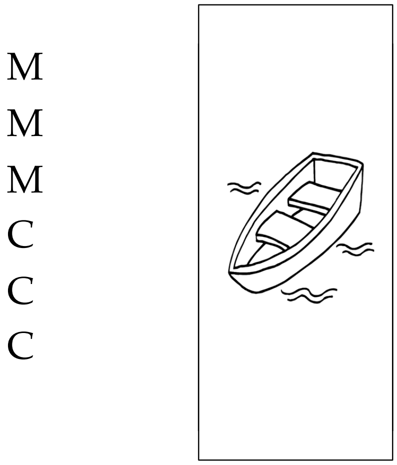
An Example

The Missionaries and Cannibals Problem:

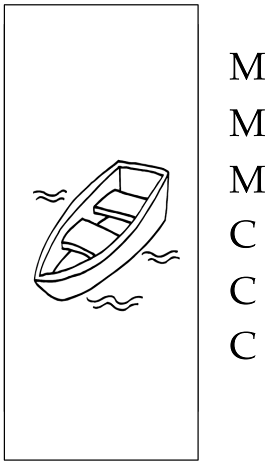
You have to transport three missionaries and three cannibals across a river. You have at your disposal a small boat that can only carry two creatures. The problem is that if the cannibals ever outnumber the missionaries on one of the riverbanks ... well you can guess what happens.

The Missionaries and Cannibal Problem:

Initial State:



Goal State:



- Operators: boat that moves two people across
- Restrictions:
  - one person must be in boat to move boat
  - can't have more cannibals than missionaries on the same side of the river (including whoever is in boat on that side)

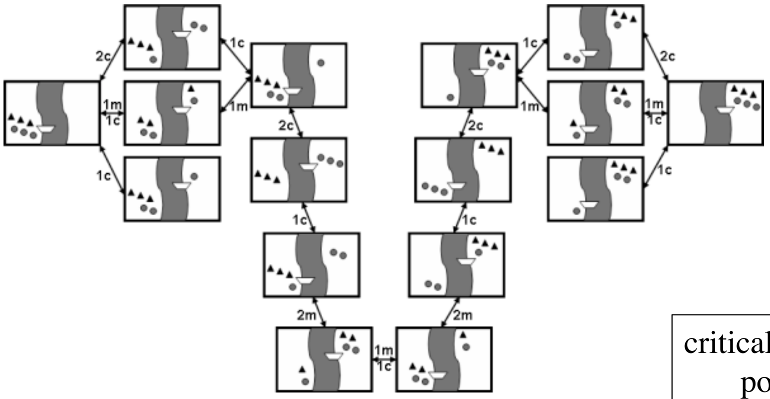
Let's try it!

http://www.plastelina.net/games/game2.html

Why is this problem hard?

- And, more specifically, why can't a simple search ("hill climbing") heuristic solve it?

The Problem Space



critical choice point!

○ cannibals
△ missionaries
1c = one cannibal moves across the river
2m = two missionaries move across the river

Choice point: must go BACK

- simple hill-climbing (always move towards goal) doesn't work….

More heuristics

Working backwards

- start from goal state
- figure out what move right before goal should be
- repeat

Working backwards

- Joe gives Nick and Tom as many peanuts as each already has. Then Nick gives Joe and Tom as many peanuts as each of them then has. Finally, Tom gives Nick and Joe as many peanuts as each has. If at the end each has sixteen peanuts, how many peanuts did each have at the beginning?

Working backwards

- Working backwards, we can start with the end result and analyze each step:
- "If at the end each has sixteen peanuts"

	Joe	Nick	Tom
Peanuts at end:	16	16	16

Working backwards

- Joe gives Nick and Tom as many peanuts as each already has. Then Nick gives Joe and Tom as many peanuts as each of them then has. Finally, Tom gives Nick and Joe as many peanuts as each has. If at the end each has sixteen peanuts, how many peanuts did each have at the beginning?

Working backwards

- “Tom gives Nick and Joe as many peanuts as each has”

	Joe	Nick	Tom
Peanuts at end:	16	16	16
Previous round:	8	8	32

Working backwards

- Joe gives Nick and Tom as many peanuts as each already has. Then Nick gives Joe and Tom as many peanuts as each of them then has. Finally, Tom gives Nick and Joe as many peanuts as each has. If at the end each has sixteen peanuts, how many peanuts did each have at the beginning?

Working backwards

- “Nick gives Joe and Tom as many peanuts as each of them then has”

	Joe	Nick	Tom
Peanuts at end:	16	16	16
Previous round:	8	8	32
Previous round:	4	28	16

Working backwards

- Joe gives Nick and Tom as many peanuts as each already has. Then Nick gives Joe and Tom as many peanuts as each of them then has. Finally, Tom gives Nick and Joe as many peanuts as each has. If at the end each has sixteen peanuts, how many peanuts did each have at the beginning?

Working backwards

- “Joe gives Nick and Tom as many peanuts as each already has.”

	Joe	Nick	Tom
Peanuts at end:	16	16	16
Previous round:	8	8	32
Previous round:	4	28	16
Peanuts at start:	26	14	8

More heuristics

Means-ends analysis

- assess current state with regards to goal
- identify biggest difference
- make that your immediate goal (“subgoal”)
- now either:
  - select operator for subgoal
  - OR
  - repeat above

- General Problem Solver: computer program that uses means-ends analysis to solve problems

---

Means-ends Example

- Problem: Get from home (NJ) to granny (LA)
- Difference-operator table

operator

difference
(between start and goal)

	airplane	**train**	**car**	**bus**	**walk**
+1000 miles	x				
100 - 1000		x	x		
1 - 100			x	x	
< 1 mile					x
	at airport	at station	in car	at bus stop	none

precondition

Means-ends Example

- Problem: Get from NJ to LA
- Difference-operator table

operator

difference (between start and goal)

	airplane	train	car	bus	walk
+1000 miles	x				
100 - 1000		x	x		
1 - 100			x	x	
< 1 mile					x
	at airport	at station	in car	at bus stop	none

precondition

Means-ends Example

- Problem: Get from NJ to LA
- Difference-operator table

operator

difference (between start and goal)

	airplane	train	car	bus	walk
+1000 miles	x				
100 - 1000		x	x		
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precondition

Means-End

	airplane	train	car	bus	walk
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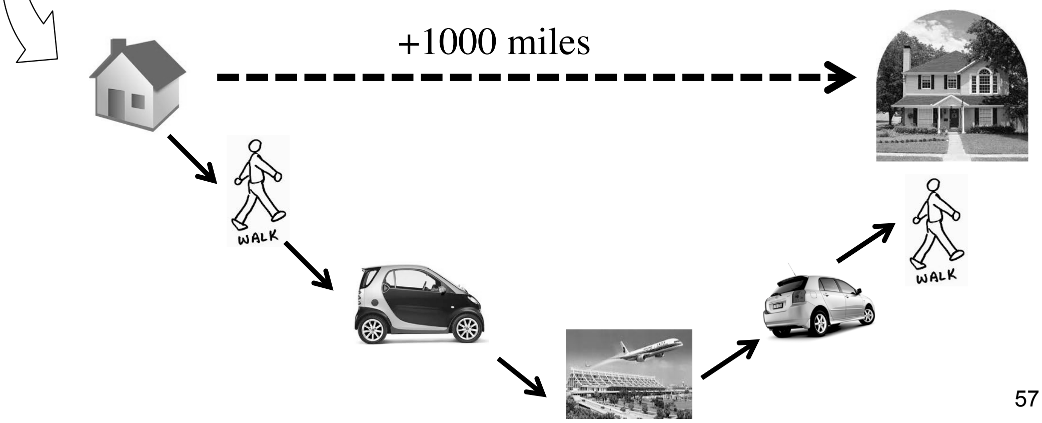


Summary of Search

- For well-defined problems, need to move through problem space by applying operators.
- At each step, must search to find best operator to apply next.
  - Can do it systematically (algorithmic), but may require too much time, too many resources.
  - Can use a heuristic (though may fail):
    - simple search
    - working backwards
    - means-ends

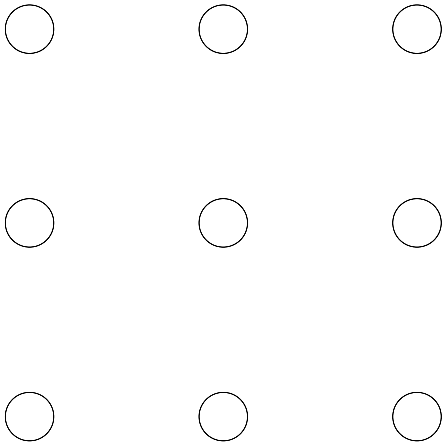
Representation

Also have to think about how to represent problem:

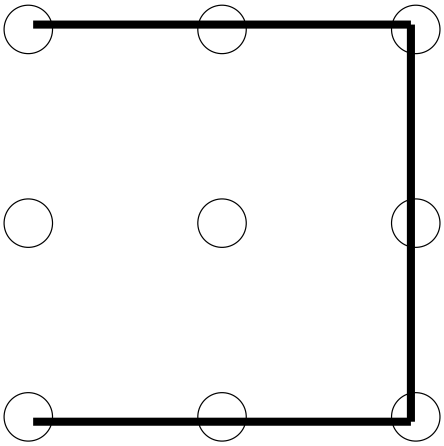
Missing, incorrect, or biased problem representation can impede problem solving ability.

- problem space can include irrelevant parts or exclude relevant parts
- operators can be missing or difficult to apply

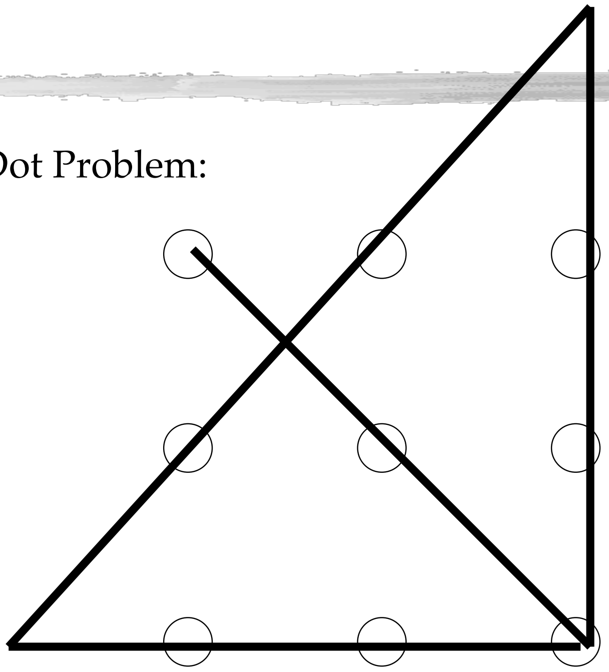
Nine Dot Problem:



Nine Dot Problem:

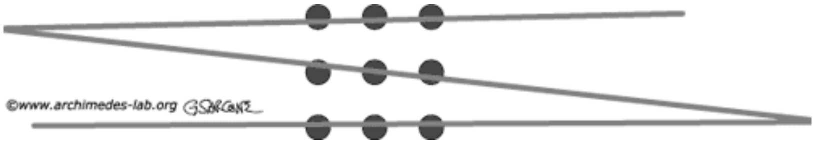


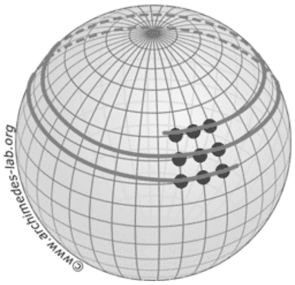
Nine Dot Problem:



Nine-dot problem

- Two more solutions….





Individual differences

- What predicts success in problem solving?

Individual differences

- Tower of Hanoi
  - Performance is thought to reflect
    - Ability to plan ahead to solve a problem
    - Frontal lobe function
  - Performance is not correlated with
    - Verbal IQ
    - Inhibition control (ability to inhibit a prepotent response, e.g., Stroop Task)
  - Performance does improve with
    - age
  - Performance is impaired in:
    - autism, Asperger syndrome, intellectual disabilities

Thought Question

In the missionaries and cannibals problem, it is generally the case that two creatures are brought over and one is then brought back, yielding a net increase in creatures on the "goal" side. In one critical step, however, **two** creatures are brought back in the boat. For each of the following search strategies, say whether a solution to this problem is:

(1) guaranteed, (2) possible but not guaranteed,

(3) impossible

a. algorithmic
b. hill climbing (simple search)
c. working backwards
d. means-ends